Progress in Studies on the Equilibrium Shape of Headland-bay Shoreline

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Abstract: Research on the laws controlling the shoreline equilibrium shape has been one important topic of studying the evolvement and stabilization of sandy coasts. After a brief review of the progress on the equilibrium shape laws research, five models are introduced in detail. Advantages and disadvantages of these models are then discussed, which leads to the conclusion that the empirical formula integrating with analysis of mechanism should be the future direction of study on the headland-bay equilibrium shape laws. Finally, the importance of the study on the equilibrium shape of headland-bay in China is also discussed.

Keywords: headland-bay; equilibrium shape model; shoreline evolution

Introduction

Coast evolvement has an important impact on the social and economic development in coastal zones. As a consequence, research on the laws controlling the shoreline equilibrium shape has been one important topic of studying the evolvement and stabilization of sandy coasts. Discovering and applying these laws makes it possible to predict the ultimate erosion and deposit status of shorelines, hence it provides critical scientific basis for coastal resources exploitation and utilization, coastal engineering, and coastline protection.

Headland-bay beach is a ubiquitous feature on both exposed and sheltered sedimentary coasts containing headlands, which represent about 51 % of the world coastlines (Short and Masselink, 1999) [1]. Sandy beaches bounded by headlands usually develop asymmetric curvature shapes under joint processes of waves and alongshore or offshore streams. According to their planar shapes, researchers have grouped them into several categories and named them correspondingly, e.g. *Zeta- curved bay* (Halligan, 1904) [2], *Half-heart shaped bay* (Silvester, 1960) [3], *Logarithmic spiral beach* (Yasso, 1965) [4], *Crenulate shape bay* (Silvester and Ho, 1972) [5], *Curved or hooked beach* (Rea and Komar, 1975) [6], *Pocket beach* (Silvester et al., 1980) [7], and so on.

Although headland-bay beaches often exhibit various shapes, they still share some common features. If taking into account the causes of beach shape formation, a headland-bay beach can be divided into two major parts (as shown in Fig.1): 1) Shadow zone, which is a cloaked region of the upper headland. Prevalent waves diffracted by the headland and refracted by the beach often erode the beach in

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this region into a concave shape; 2) Tangential zone, which is a straight line region of the lower coast. Shoreline in this region is often parallel to prevalent wave fronts (Silvester, 1974) [8].

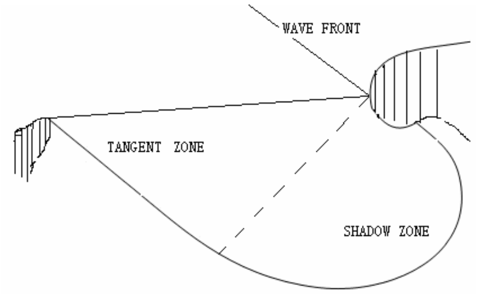

Fig. 1 The structure of headland-bay beach

In addition to the shape difference, the stabilities of sandy beaches are also different. In terms of beach stability, headland-bay beaches can be classified as *Static equilibrium beaches, Dynamic equilibrium beaches*, and *Unstable beaches* (Hsu, 2000) [9]. When a beach reaches its static equilibrium status, waves approach perpendicularly to the shoreline, breaking simultaneously. Consequently, there is no alongshore component of the incoming wave energy, which leads to an almost zero alongshore sediment transporting. At this stage, there is no long-term erosion or deposition within the embayment. However, if a beach reaches its dynamic equilibrium status, there will be a certain angle of the breaking waves approaching the shoreline. This will induce alongshore currents and alongshore drift, at the same time, there could be alongshore sediment transported from the upper coast or river to the lower coast. Littoral drift and sediment supply may reach a dynamic balance, hence maintaining a dynamic stable shoreline. When conditions of a dynamic equilibrium bays change, such as disturbance to the continuity of littoral drift, wave sheltered by coastal structures and reduction in the sediment supply from river, the balance between littoral drift and sediment supply will be broken and beaches will be eroded or accumulated. It is at this time that the beach is in its unstable stage. Because a headland-bay beach in static equilibrium was suggested as the most stable one, such a beach's structure has been recommended as the ultimate form of a stable shoreline.

1 Laws of coast evolvement and the proposing of the planar equilibrium shape model

Earlier studies on coastline shapes mainly focused on explaining their evolvement mechanisms in the view of underlying driving forces (Johnson, 1919; Davies, 1958, 1960) [10-12]. Under impacts of waves, coast will suffer erosion and retreat toward inland. If only the long-term change in erosion and accumulation is considered and the seasonal beach surface adjustment is ignored, change in shoreline position can be calculated from the sediment transporting equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{1}{d}\frac{\mathrm{d}S}{\mathrm{d}x} \qquad (1)$$

Where, S is the alongshore sand-transport. Sand is assumed to be transported alongshore by the action of breaking waves. The empirical formula for the alongshore sand-transport is (Komar, 1985)[13]:

$$S = K(EC_n)_b \sin\alpha_b \cos\alpha_b \qquad (2)$$

Equation 1 indicates that the change rate of shoreline position (dy/dt) is determined by the alongshore change rate of sand-transport (dS/dx). If dS/dx is positive, which indicates that the sand-transport increases along x, then dy/dt is negative, which means that shoreline will be eroded and will back off. Otherwise, if dS/dx is negative, dy/dt will be positive, which indicates that there will be sediment accumulated on the shore. On the other hand, if S remains constant, then dS is 0 and dy/dt is also 0, shoreline is stable.

Based on Eq. 1 and Eq. 2, theoretically, the shoreline location can be obtained at anytime. However, because the factors controlling the alongshore sand-transport S are very complicated, it is usually impossible to obtain an analytical resolution of Eq.1. On the other hand, numerical resolutions often induce large errors at big temporal and spatial scales. For these reasons, some researchers have proposed several empirical functions to simulate the shoreline planar form of headland-bay beaches through studying numbers of prototype beaches and laboratory simulations.

2 Equilibrium shape models of headland-bay beaches

Jennings first recognized the crenulate-shaped bay as a stable physiographic feature in 1955 (Jennings, 1955) [14]. Davies discovered the importance of wave refraction (Davies, 1958) [11]. Le Blond derived equations for littoral drift and showed that the resulting shoreline became a spiral shape in the shadow zone of the upper coast headland, but could not explain why (Le Blond, 1972) [15]. But, later he confirmed that result by calculating the arrival time of waves diffracting around the headland as shallow water waves breaking simultaneously around the periphery (Le Blond, 1979) [16]. Yasso proposed the logarithmic spiral model at first through fitting a lot of prototype bays .

2.1 Logarithmic spiral model

The logarithmic spiral bay shape equation is:

$$R_2 / R_1 = \exp(\theta \cot\alpha) \qquad (3)$$

Where, θ is the angle between R_1 and R_2, which are polar radii; α is the logarithmic spiral parameter, which is confirmed by obliquity of the incident wave, β. The coordinate systems are polar coordinates, where the upper coast headland B is apex and the axis is parallel to the wave front (as shown in Fig. 2).

The proposing of logarithmic spiral model provided a new way to study laws of equilibrium shoreline shapes. Results form the Logarithmic spiral model match well with prototype bays in the shadow zone of the upper coast headland, whereas, they deviate from prototype bays at the lower coast end and the smaller the β, the larger the deviation (Hsu and Silvester, 1989) [17]. Rea and Komar commented that to make the modeled curve match well with the prototypes, the theoretical center derived from the model must be shifted (Rea and Komar, 1975) [6]. Xia pointed out that not only its centre didn't match the point where diffraction takes place, but also there was large deviation between the logarithmic spiral curve shape and that of those prototypes. For these reasons, and maybe others, it was difficult or even impossible to apply the model in real life (Xia, 1988) [18].

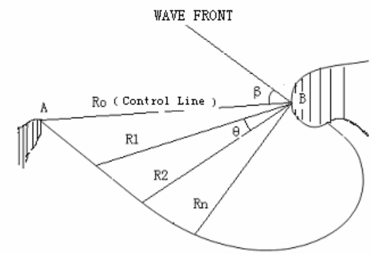

Fig. 2 The model factor of equilibrium beach

2.2 Shape-factors relation model

Comparing the shape factors of some prototype and model bays, Hus, Silvester and Xia obtained function curves that satisfied the relationship between shape factors and plotted them in diagrams. These curves serve as criteria to tell the static equilibrium status of a beach.

2.2.1 R/R_0 and θ Relation model

Polar coordinate system is used here. Upper coast headland, where wave diffraction happens, is the polar original point and the wave front is the polar axis. R_0 is the length of control line, R is the discretional polar radii, θ is the corresponding polar angle with R. Applying shape-factors data of prototype and model bays, functional relation curve of R/R_0 and θ can be obtained for the corresponding obliquity of the incident wave β (Fig.3).

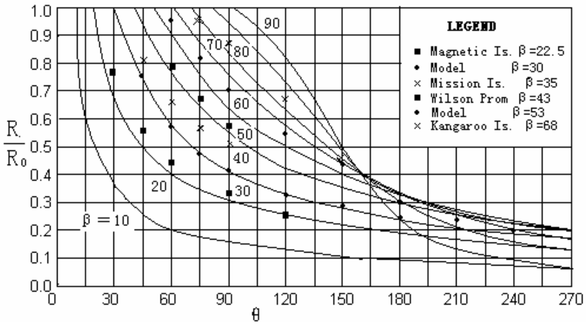

Fig. 3 Arc ratio R/R_0 versus radius angle θ for specific obliquity

The data extended over β =22.5° to 72°, which covers most natural bays. The lines have been extrapolated down to β =10° and up to β =90° in the model. We can apply it as the criterion of stability. If a beach's shape factors fall on the curve from the model, the bay is an equilibrium bay, otherwise it is unstable.

2.2.2 a/R_0 and β relation model

a is the largest bay indenting ratio, i.e. the largest distance from the control line to the bay. β is the obliquity of the incident wave. Their functional relationship was showed in Fig. 4. Similarly, if a beach's shape factors fall on the curve from the model, the bay is an equilibrium bay, otherwise, if they fall under the curve, the bay is being eroded. Dai applied the criterion to test the stability of 34 headland bays in South China (Dai, 2004) [19].

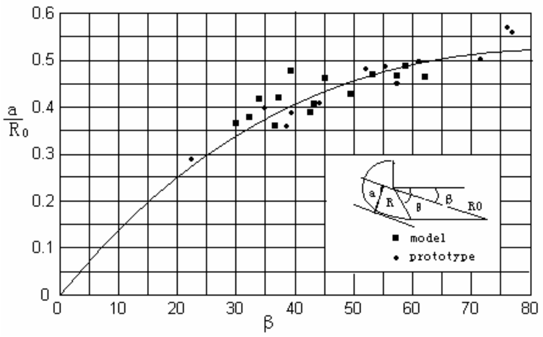

Fig. 4 Indentation Ratio a/R_0 versus Obliquity β

2.3 Hyperbolic model

Through experiments on the formation of equilibrium bays, Xia found that the shapes of equilibrium bays were completely alike among all cases with different incident wave angles. By analyzing the experimental data, Xia obtained a new static equilibrium shoreline function (Xia, 1988) [18]:

$$R^m \cdot \theta = K , \quad \text{or} \quad (R/a)^m \cdot (\theta/\beta) = 1 \tag{4}$$

Where, $m=\sqrt{2}$ (for non-static equilibrium shoreline, $m<\sqrt{2}$), R is a discretional polar radii, θ is the corresponding polar angle of R (as shown in Fig. 2). K is constant and can be expressed as:

$$k = a^m \cdot \beta$$

Where, R_0 is the length of control line, β is the obliquity of the incident wave. In the Cartesian system of coordinates, R and θ show a hyperbolic relation.

Through analyzing the data of 5 experimental bays and 9 natural bays, Xia confirmed the law. Moreover, he discussed the application of this model in the headland protection engineering (xia, 1991, 1994) [20, 21].

2.4 Parabolic model

Hsu and Evans have developed a second-order polynomial equation (namely parabolic equation) to fit the planar forms of 27 mixed cases of prototype and model bays believed to be in static equilibrium (Hsu and Evans, 1989) [17]. The equation can be expressed as:

$$R_n/R_0 = C_1 + C_2 \cdot \left(\beta/\theta\right) + C_3 \cdot \left(\beta/\theta\right)^2 \qquad (5)$$

Where R_n is the discretional polar radii, θ is the corresponding polar angle of R , R_0 is the length of control line, β is the obliquity of the incident wave (as shown in Fig.2). C_1, C_2 and C_3, generated by regression analysis to fit the 27 prototypes and model bays mentioned previously, vary with the angle β (The range of angle β from 10° to 80°, which covers most natural bays). These coefficients may be expressed by fourth-order polynomials as follows:

$$C_1 = 0.0707 - 0.0047\beta + 0.000349\beta^2 - 0.00000875\beta^3 + 0.00000004765\beta^4 \qquad (6)$$

$$C_2 = 0.9536 + 0.0078\beta - 0.00004879\beta^2 + 0.0000182\beta^3 - 0.000001281\beta^4 \qquad (7)$$

$$C_3 = 0.0214 - 0.0078\beta + 0.0003004\beta^2 - 0.00001183\beta^3 + 0.00000009343\beta^4 \qquad (8)$$

Subsequently, Silvester and Hsu produced necessary verification on this equation (Silvester and Hsu, 1993, 1997) [22, 23]. The parabolic bay shape equation has been cited in the *coastal Engineering Manual (2002)* for coastal sediment processes and shore protection projects.

Based on the parabolic model, Mauricio demonstrated the location of lower coast headland and extended the application of the parabolic model (Mauricio, 2001) [24].

2.5 Elliptic model

Prediction of the shoreline response to detached breakwaters is an important topic of studying the shoreline planar form. Detached breakwaters are the costal protecting construction parallel to and far away from shoreline. Their primary function is to protect shoreline by making wave energy dissipate and sediment accumulates. Because the hydrodynamic mechanism and sediment transport are extremely complicated along the detached breakwaters, most researchers have proposed several empirical formulas to simulate the equilibrium shoreline (Shinohara and Tsubaki, 1966, Rosen and Vajada, 1982, Mimura, Shimizu and Horikawa, 1983, Hsu and Silvester, 1990, McCormick, 1993, Ming and Chiew, 2000) [25-30].

McCormick’s elliptic model is a typical one. In a Cartesian coordinate system having the origin at the center of breakwater, the equation for the shoreline is:

$$\frac{(y \mp h)^2}{a^2} + \frac{x^2}{b^2} = 1 \qquad (9)$$

Where h is the distance from the center of the breakwater to the center of the ellipse, a is the semimajor axis, and b is the semiminor axis (as shown in Fig.5).

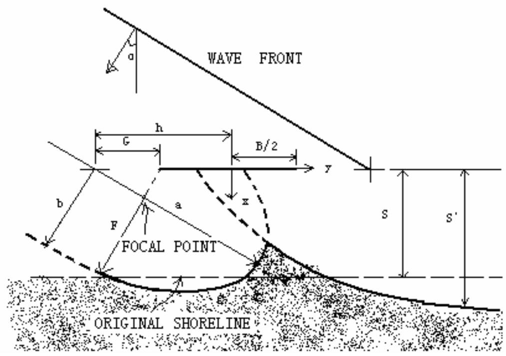

Fig.5 Elliptical shoreline response to single detached breakwater

The values of h, a, and b are assumed to depend on the deep-water wave steepness H_0/L_0, the beach slope S_b, the distance from the shoreline to the centerline of the breakwater S, and the breakwater length B. Here come the empirical equations:

$$\frac{b}{S} = 1 + 0.2\zeta_0 \sin(\chi\zeta_0) \tag{10}$$

$$\zeta = (H_0 / L_0) / S_b \tag{11}$$

$$\chi = -1.92(\frac{S}{B})^2 + 9.92(\frac{S}{B}) \tag{12}$$

$$a = \sqrt{G^2 + b^2} \tag{13}$$

$$h = G + 0.5B \tag{14}$$

$$\frac{G}{b} = \exp[\ln(\mu) + \sigma\ln(\zeta_0) - \nu\zeta_0] \tag{15}$$

$$\ln(\mu) = 19.4\tanh(0.91\frac{S}{B}) \tag{16}$$

$$\nu = 20.0\tanh(0.99\frac{S}{B}) \tag{17}$$

$$\sigma = 17.0\tanh(0.59\frac{S}{B}) \tag{18}$$

Combining Eq.9 and Eq.18, an equilibrium shoreline can be obtained. McCormick discussed the shoreline response to multiple detached breakwaters. To demonstrate its utility, the shoreline response to an 11-unit breakwater system at Bay Ridge, Md., was predicted. The predicted and actual shorelines were shown to be in good agreement.

Hsu found more parameters having significant effects on the shoreline and modified the elliptic model (Hsu, 2003) [31].

3 Conclusions

The proposing and developing of the empirical models on the headland-bay beach planar equilibrium shape have provided the research community new thoughts and research directions to study laws of beach planar equilibrium shape. It's been almost half century since the very first Logarithmic Spiral model came into being, and models on the beach planar equilibrium shape have also evolved into their mature stage. These models not only theoretically provide new ideas and methods for researches on beaches, but also are very useful in practical applications. Predictions of static equilibrium coastlines based on models makes it is possible to avoid regions under possible erosion risks during deployment of civilian constructions in coastal zones. At the same time, empirical models on the beach equilibrium shape, especially the Parabolic model, have been widely applied in coastal engineering and have become powerful theoretical tools for coastal engineers to design economical, reasonable, and efficient coastal protection projects such as headland controlling projects, detached breakwater projects, and so forth.

However, empirical models were based on the statistical fitting; hence lack mechanism of action explanation. Although the mechanism of along-shore sediment transport has explained the mechanism of the formation of shoreline, confirmed the relationship between the shape of bay beach and the along-shore gradient of broken wave energy, stated the static equilibrium beach's characteristic that net along-shore sediment transport is zero when waves approach the shoreline perpendicularly and break simultaneously, there is still no explanation on why beaches agree with empirical equations, that is there is no mechanical explanation on why the coastline satisfies the logarithmic spiral, the 2nd order parabolic functional relations. In a word, models lack of theoretical basis. Linking of mechanical analysis and empirical fitting must be the future research direction of studying laws on the planar equilibrium shape of bay beaches.

To date, there have been relative fewer researches on studying laws on the planar equilibrium shape of bay beaches. In addition to Xia's parabolic model, Chang discussed factors that affect the formation of beaches by waves through conducting the wave slot experiment (Chang, 1994) [32]; Wang explored that actions of dominant and 2nd dominant waves can lead to double logarithmic spirals under the condition of monsoon (Wang, 1997) [33]. Headland-bay sandy beaches widely distribute in China, especially in southern China coasts. Recently, due to natural and human generated factors, sandy beaches is experiencing erosion and retreating. This not only provides suitable conditions for research of laws on the planar equilibrium shape of bay beaches, but also brings forward new challenges. It can be optimistic to foresee that studying laws on the planar equilibrium shape of bay beaches in China must get more and more attention from beach researchers.

Acknowledgements

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References

[1] Short A D, Masselink G. Embayed and structurally controlled beaches [M]. Handbook of Beach and Shoreface Morphodynamics, 1999, 230-249.

[2] Halligan G H. Sand movement on the New South Wales coast [J]. Proc. Limnology Soc. New South Wales, 1906, 619-640.

[3] Silvester R. Stabilization of sedimentary coastlines [J]. Nature, 1960, 467-469.

[4] Yasso W E. Plan geometry of headland bay beaches [J]. Geology, 73. 1965, 702-714.

[5] Silvester R, Ho S K. Use of crenulate shaped bays to stabilize coasts. Proc, 13th Inter. Conf. Coastal Eng., ASCE [C]. 1972, 1 394-1 406.

[6] Rea C C, Komar P D. Computer simulation models of hooked beach shoreline configuration [J]. Sedimentary Petrology, 1975, 866-872.

[7] Silverster R, Tsuchiya Y, Shibano T. Zeta bays, pocket beaches and headland control. Proc, 17th Inter. Conf. Coastal Eng.,ASCE [C],1980,1 306-1 319.

[8] Silvester R. Coastal Engineering, Ⅱ [M]. Elsevier scientific publishing company, 1974, 71-83.

[9] Hsu J R C, Uda T. Silvester R. Shoreline protection methods Japanese experience [M]. Handbook of Coastal Engineering，New York: McGraw-Hill, 2000, 9.1-9.77.

[10] Johnson D W. Shore processes an shoreline development [M]. New York: Jone Wiley & Sons, 1919.584.

[11] Davies J L. Wave refraction and the evolution of shoreline curves [J]. Geor.Stud.,1958, 1-4.

[12] Davies J L. Beach alignment in shouthern Australia [J]. Aust.Geogr., 1960, 42-44.

[13] Komar P D. Beach processes and sedimentation. Englewood Cliffs, New Jersey: Prentice-Hall, 1976.

[14] Jenning J N. The influence of wave action on coastal outline in plan [J]. Austral.Geogr.,1955, 36-44.

[15] Le Blond P H. On the formation of spiral beaches [A]. Proc. 13th Inter. Conf. Coastal Eng. 1972,1 331-1 345.

[16] Le Blond P H. An explanation of the logarithmic spiral plan shape of headland-bay beaches [J]. J.Sedi. Petrol., 1979, 1 093-1 100.

[17] Hsu J R C, Silvester R. Member, et al. Static equilibrium bays: New relationships [J]. Journal of Waterway, Port, Coastal and Ocean Engineering. 1989, 285-298.

[18] Xia YiMing, equilibrium shape law of sandy coast [R], Nanjing Hydraulic Research Institute,1988.

[19] Dai Z J, Li C C, Zhang Q L. Fractal analysis of shoreline patterns for crenulate-bay beaches [J], Southern China, Estuarine, Coastal and Shelf Science, 2004, 65-71.

[20] Xia YiMing, study on the engineering means for coastal stability [J], The Ocean Engineering, 1991, 9(4):45-58.

[21] Xia YiMing, Headland controlling projects and equilibrium shape law [J], Journal of Hohai University, 1994, 22(1): 21-32.

[22] Silester R , Ho S K. Coastal Stabilization: Innovative Concepts [M]. Englewood Cliffs, New Jersey: Prentice

Hall,1993,578.

[23] Silvester R, Hsu J R C. Coastal Stabilization [M]. Singapore. World Scientific,1997 578.

[24] Mauricio González, Raul Mendina. On the application of static equilibrium bay formulations to natural and man-made beaches [J]. Coastal Engineering, 2001, 209-225.

[25] Shinohara, K., Tsubaki, T., Model study on the change of shoreline of sandy beach by the offshore breakwater [A]. Proceedings of the tenth International Conference on Coastal Engineering[C], ASCE, 1966, 550–563.

[26] Rosen, D.S., Vajada, M., Sedimentlogical influences of detached breakwaters[A]. Proceedings of the Eighteen Conference on Coastal Engineering [C], ASCE, (3), 1982, 130-1 940.

[27] Mimura N, Shimizu T, Horikawa K. Laboratory study on the influence of detached breakwater on coastal change [A]. Proc., Coast. Struct. [C]. ASCE, Reston, VA 83, 1983, 740-752.

[28] Hsu J R C, Silvester R, Accretion behind single offshore breakwater [J]. Journal of Waterway, Ports, Coastal and Ocean Engineering, ASCE 1990, 116 (3), 362–379.

[29] McCormick M E. Equilibrium shoreline response to breakwater [J]. Journal of Waterway, Ports, Coastal, and Ocean Engineering, ASCE 1993, 119 (6), 657–670.

[30] Ming D, Chiew Y M. Shoreline changes behind detached breakwater [J]. Journal of Waterway, Ports, Coastal and Ocean Engineering, ASCE 2000, 126 (2), 63–70.

[31] Hsu T W, Jan C D, Wen C C. Modified McCormicks model for equilibrium shorelines behind a detached breakwater [J]. Ocean Engineering, 2003, 30:1 887-1 897.

[32] Chang Reifang. The experiment research on the formation of the steady bay effected by the wave [J]. Journal of Ocean University of Qingdao, 1994, special: 62-68.

[33] Wang Wenjie. On Geomorphic-Sedimentary Features and Evolution of Spiral Bay Beaches in South China [A]. Symposium on Coastal Ocean Resources and Environment ’97, 1997, Hongkong.

岬间海湾平面平衡形态研究进展

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摘　要：岬间海湾平面平衡形态规律的研究是砂质海岸稳定与演变研究的重要内容。简要回顾了岬间海湾平面形态规律的研究进展，着重介绍了研究海湾平衡形态的五个模型。通过评述平衡形态模型的优缺点，指出机理分析和经验拟合相结合应该为以后海湾平面平衡形态规律的研究发展方向，并论述了我国岬间砂质海湾平面平衡形态研究的重要意义。

关键词：岬间海湾；平衡形态模型；海岸演变